

# TRADING VOL WITHOUT OPTIONS

## Guide to volatility swaps / variance swaps / gamma swaps, options on variance and futures on VIX / vStoxx

**Colin Bennett**  
Head of Derivatives Strategy  
(+34) 91 289 3056  
cdbennett@gruposantander.com

**Miguel A. Gil**  
Equity Derivatives Strategy  
(+34) 91 289 5515  
mgil@gruposantander.com

While volatility trading has historically been performed by delta hedging options, a variety of alternative products have been created to make volatility trading easier. We examine the differences and similarities between volatility swaps, gamma swaps and variance swaps. The pricing of related products, such as corridor variance swaps, options on variance and futures on volatility indices (VIX and vStoxx), are also analysed.

- **VOLATILITY SWAPS:** A theoretical drawback of volatility swaps is the fact that they require a volatility of volatility (as they are short vol of vol) model for pricing, as options need to be bought and sold during the life of the contract (which leads to higher trading costs). However, in practice, the vol of vol risk is small and volatility swaps trade roughly in line with ATM forward (ATMf) implied volatility.
- **VARIANCE SWAPS:** As variance swaps can be replicated by delta hedging a static portfolio of options, it is not necessary to buy or sell options during the life of the contract. The problem with this replication is that it assumes options of all strikes can be bought, but in reality very OTM options either are not listed or are not liquid. Selling a variance swap and only hedging with the available, roughly ATM, options leaves the vendor short tail risk. As the payout is on variance, which is volatility squared, the amount can be very significant. We also examine the payout of corridor variance swaps.
- **GAMMA SWAPS:** Gamma swaps are of interest to dispersion traders, as dispersion can be implemented via a static portfolio of gamma swaps (hence it is easier to maintain than a dispersion trade using variance swaps, which has to be rebalanced).
- **OPTIONS ON VARIANCE SWAPS:** As the liquidity of the variance swap market improved in the mid 2000's, market participants started to trade options on variance. As volatility is more volatile at high levels, skew is positive (the inverse of the negative skew seen in the equity market). In addition, volatility term structure is inverted, as volatility mean reverts and does not stay elevated for long periods.
- **FUTURE ON VOLATILITY INDEX (VIX/VSTOXX):** Recently, futures on the VIX and vStoxx have become more liquid due to increased structured product activity on them. While the calculation of these volatility indices is similar to a variance swap calculation, as the payout is based on the square root of variance their payout is linear in volatility not variance. They are, therefore, short vol of vol, just like volatility swaps.

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## VOLATILITY, VARIANCE AND GAMMA SWAPS

In theory, the profit and loss from delta hedging an option is fixed and is based solely on the difference between the implied volatility of the option when it was purchased and the realised volatility over the life of the option. In practice with discrete delta hedging and unknown future volatility, this is not the case. That led to the creation of volatility, variance and gamma swaps. These products also remove the need to continuously delta hedge, which can be very labour-intensive and expensive. Until the credit crunch, variance swaps were the most liquid of the three but, now, volatility swaps are more popular for single stocks.

### VOLATILITY, VARIANCE & GAMMA SWAPS GIVE PURE VOL EXPOSURE

**Naming of volatility swaps, variance swaps and gamma swaps is misleading as they are in fact forwards**

As spot moves away from the strike of an option the gamma decreases, and it becomes more difficult to profit via delta hedging. Second-generation volatility products such as volatility swaps, variance swaps and gamma swaps were created to give volatility exposure for all levels of spot and also avoid the overhead and cost of delta hedging. While volatility and variance swaps have been traded since 1993, they became more popular post 1998 when Russia defaulted on its debts and Long-Term Capital Management (LTCM) collapsed. The naming of volatility swaps, variance swaps and gamma swaps is misleading as they are in fact forwards. This is because their payoff is at maturity, whereas swaps have intermediate payments.

- **Volatility swaps:** Volatility swaps were the first product to be traded significantly and became increasingly popular in the late 1990s until interest migrated to variance swaps. Following the collapse of the single stock variance market in the credit crunch, they are having a renaissance due to demand from dispersion traders. A theoretical drawback of volatility swaps is the fact that they require a volatility of volatility (vol of vol) model for pricing, as options need to be bought and sold during the life of the contract (which leads to higher trading costs). However, in practice the vol of vol risk is small and volatility swaps trade roughly in line with ATM forward (ATMf) implied volatility.
- **Variance swaps:** The difficulty in hedging volatility swaps drove liquidity towards the variance swap market, particularly during the 2002 equity collapse. As variance swaps can be replicated by delta hedging a static portfolio of options, it is not necessary to buy or sell options during the life of the contract. The problem with this replication is that it assumes options of all strikes can be bought, but in reality very OTM options either are not listed or are not liquid. Selling a variance swap and only hedging with the available roughly ATM options leaves the vendor short tail risk. As the payout is on variance, which is volatility squared, the amount can be very significant. For this reason, liquidity on single stock variance disappeared in the credit crunch.
- **Gamma swaps:** Dispersion traders profit from overpriced index implied volatility by going long single stock variance and short index variance. The portfolio of variance swaps is not static, hence rebalancing trading costs are incurred. Investment banks attempted to create a liquid gamma swap market, as dispersion can be implemented via a static portfolio of gamma swaps (and, hence, it could better hedge the exposure of their books from selling structured products). However, liquidity never really took off due to limited interest from other market participants.



## VOLATILITY SWAP ≤ GAMMA SWAP ≤ VARIANCE SWAP

Variance swaps are quoted as the square root of variance (to allow easier comparison with implied volatility)

Variance and gamma swaps are normally quoted as the square root of variance to allow easier comparison with the options market. However, typically variance swaps trade in line with the 30 delta put (if skew is downward sloping as normal). The square root of the variance strike is always above volatility swaps (and ATMf implied as volatility swaps  $\approx$  ATMf implied). This is due to the fact a variance swap payout is convex (hence will always be greater than or equal to volatility swap payout of identical vega, which is explained later in the section). Only for the unrealistic case of no vol of vol (ie, future volatility is constant and known) will the price of a volatility swap and variance swap (and gamma swap) be the same. The fair price of a gamma swap is between volatility swaps and variance swaps.

### (1) VOLATILITY SWAPS

Only for flat skew will the price of a volatility swap and variance swap (and gamma swap) be the same

The payout of a volatility swap is simply the notional, multiplied by the difference between the realised volatility and the fixed swap volatility agreed at the time of trading. As can be seen from the payoff formula below, the profit and loss is completely path independent as it is solely based on the realised volatility. Volatility swaps were previously illiquid, but are now more popular with dispersion traders, given the single stock variance market no longer exists post credit crunch. Unless packaged as a dispersion, volatility swaps rarely trade. As dispersion is short index volatility, long single stock volatility, single stock volatility swaps tend to be bid only (and index volatility swaps offered only).

#### *Volatility Swap Payoff*

$$(\sigma_F - \sigma_S) \times \text{volatility notional}$$

where:

$\sigma_F$  = future volatility (that occurs over the life of contract)

$\sigma_S$  = swap rate volatility (fixed at the start of contract)

Volatility notional = Vega = notional amount paid (or received) per volatility point

### (2) VARIANCE SWAPS

Variance swaps are identical to volatility swaps except their payout is based on variance (volatility squared) rather than volatility. Variance swaps are long skew (more exposure to downside put options than upside calls) and convexity (more exposure to OTM options than ATM). One-year variance swaps are the most frequently traded.

#### *Variance Swap Payoff*

$$(\sigma_F^2 - \sigma_S^2) \times \text{Variance notional}$$

where:

Variance notional = notional amount paid (or received) per variance point

NB: Variance notional = Vega / (2 ×  $\sigma_S$ ) where  $\sigma_S$  = current variance swap price

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## VARIANCE SWAPS CAPS ARE EFFECTIVELY SHORT OPTION ON VAR

Variance swaps on single stocks and emerging market indices are normally capped at 2.5 times the strike, in order to prevent the payout from rising towards infinity in a crisis or bankruptcy. A cap on a variance swap can be modelled as a vanilla variance swap less an option on variance whose strike is equal to the cap. More details can be found in the section OPTIONS ON VARIANCE.

### *Capped Variance Should be Hedged With OTM Calls, Not OTM Puts*

The presence of a cap on a variance swap means that if it is to be hedged by only one option it should be a slightly OTM call, not an OTM (approx delta 30) put. This is to ensure the option is so far OTM when the cap is hit that the hedge disappears. If this is not done, then if a trader is long a capped variance swap he would hedge by going short an OTM put. If markets fall with high volatility hitting the cap, the trader would be naked short a (now close to ATM) put. Correctly hedging the cap is more important than hedging the skew position.

### *S&P500 Variance Market Is Increasing in Liquidity, While SX5E Has Become Less Liquid*

The payout of volatility swaps and variance swaps of the same vega is similar for small payouts, but for large payouts the difference becomes very significant due to the quadratic (ie squared) nature of variance. The losses suffered in the credit crunch from the sale of variance swaps, particularly single stock variance (which, like single stock volatility swaps now, was typically bid), have weighed on their subsequent liquidity. Now variance swaps only trade for indices (usually without cap, but sometimes with). The popularity of VIX futures has raised awareness of variance swaps, which has helped S&P500 variance swaps become more liquid than they were before the credit crunch. S&P500 variance swaps now trade with a bid offer spread of c30bp and sizes of approximately US\$5mn vega can regularly trade every day. However, SX5E variance swap liquidity is now a fraction of its pre-credit-crunch levels, with bid-offer spreads now c80bp compared with c30bp previously.

## CORRIDOR VARIANCE SWAPS ARE NOT LIQUID

**Corridor variance swaps give exposure to volatility, only when spot is in a certain range**

As volatility and spot are correlated, volatility buyers would typically only want exposure to volatility levels for low values of spot. Conversely volatility sellers would only want exposure for high values of spot. To satisfy this demand, corridor variance swaps were created. These only have exposure when spot is between spot values A and B. If A is zero then it is a down variance swap. If B is infinity it is an up variance swap. There is only a swap payment on those days the spot is in the required range, so, if spot is never in the range there is no payment. Because of this, a down variance swap and up variance swap with the same spot barrier is simply a vanilla variance swap. The liquidity of corridor variance swaps was always far lower than for variance swaps and, since the credit crunch, they are rarely traded.

### *Corridor Variance Swap Payoff*

$(\sigma_{F \text{ when in corridor}}^2 - \sigma_S^2) \times \text{variance notional} \times \text{percentage of days spot is within corridor}$

where:

$\sigma_{F \text{ when in corridor}}$  = future volatility (of returns  $P_i/P_{i-1}$  which occur when  $B_L < P_{i-1} \leq B_H$ )  
 $B_L$  and  $B_H$ , are the lower and higher barriers, where  $B_L$  could be 0 and  $B_H$  could be infinity.



### (3) GAMMA SWAPS

The payout of gamma swaps is identical to that of a variance swap, except the daily P&L is weighted by spot ( $\text{price}_n$ ) divided by the initial spot ( $\text{price}_0$ ). If spot range trades after the position is initiated, the payouts of a gamma swap are virtually identical to the payout of a variance swap. Should spot decline, the payout of a gamma swap decreases. Conversely, if spot increases, the payout of a gamma swap increases. This spot-weighting of a variance swap payout has the following attractive features:

**Gamma swaps are ideal for trading dispersion as it is “fire and forget”**

- Spot weighting of variance swap payout makes it unnecessary to have a cap, even for single stocks (if a company goes bankrupt with spot dropping close to zero with very high volatility, multiplying the payout by spot automatically prevents an excessive payout).
- If a dispersion trade uses gamma swaps, the amount of gamma swaps needed does not change over time (hence the trade is ‘fire and forget’ as the constituents do not have to be rebalanced as they would if variance swaps were used).
- A gamma swap can be replicated by a static portfolio of options (although a different static portfolio to variance swaps), which reduces hedging costs. Hence, no volatility of volatility model is needed (unlike volatility swaps).

#### *Gamma Swap Market Has Never Had Significant Liquidity*

A number of investment banks attempted to kick start a liquid gamma swap market, partly to satisfy potential demand from dispersion traders and partly to get rid of some of the exposure from selling structured products (if the product has less volatility exposure if prices fall, then a gamma swap better matches the change in the vega profile when spot moves). While the replication of the product is as trivial as for variance swaps, it was difficult to convince other market participants to switch to the new product and liquidity stayed with variance swaps (although after the credit crunch, single stock variance liquidity moved to the volatility swap market). If the gamma swap market ever gains liquidity, long skew trades could be put on with a long variance-short gamma swap position (as this would be long downside volatility and short upside volatility as a gamma swap).

#### *Gamma Swap Payoff*

$$(\sigma_G^2 - \sigma_S^2) \times \text{variance notional}$$

where:

$$\sigma_G^2 = \text{future spot weighted (i.e. multiplied by } \frac{\text{price}_n}{\text{price}_0} \text{) variance}$$

$$\sigma_S^2 = \text{swap rate variance (fixed at the start of contract)}$$

## PAYOUT OF VOLATILITY, VARIANCE AND GAMMA SWAPS

The payout of volatility swaps, variance swaps and gamma swaps is the difference between the fixed and floating leg, multiplied by the notional. The calculation for volatility assumes zero mean return (or zero drift) to make the calculation easier and to allow the variance calculation to be additive.

- **Fixed leg:** The cost (or fixed leg) of going long a volatility, variance or gamma swap is always based on the swap price,  $\sigma_S$  (which is fixed at inception of the contract). The fixed leg is  $\sigma_S$  for volatility swaps, but is  $\sigma_S^2$  for variance and gamma swaps).
- **Floating leg:** The payout (or floating leg) for volatility and variance swaps is based on the same variable  $\sigma_F$  (see equation below). The only difference is that a volatility swap payout is based on  $\sigma_F$  whereas for a variance swap it is  $\sigma_F^2$ . The gamma swap payout is based on a similar variable  $\sigma_G^2$ , which is  $\sigma_F^2$  multiplied by  $\text{price}_n/\text{price}_0$ .

$$\sigma_F = 100 \times \sqrt{\frac{\sum_{n=1}^N [\text{Ln}(\text{return}_n)]^2}{N_{\text{exp}}} \times \text{number business days in year}}$$

$$\sigma_G = 100 \times \sqrt{\frac{\sum_{n=1}^N \frac{\text{price}_n}{\text{price}_0} [\text{Ln}(\text{return}_n)]^2}{N_{\text{exp}}} \times \text{number business days in year}}$$

$$\text{return}_n = \frac{\text{price}_n}{\text{price}_{n-1}} \text{ for indices}$$

$$\text{return}_n = \frac{\text{price}_n + \text{dividend}_n}{\text{price}_{n-1}} \text{ for single stocks (dividend}_n \text{ is dividend going ex on day } n)$$

where:

number of business days in year = 252 (usual market practice)

$N_{\text{exp}}$  = Expected value of N (if no market disruption occurs). A market disruption is when shares accounting for at least 20% of the index market cap have not traded in the last 20 minutes of the trading day.



**As variance is additive, payout is not path dependent and no vol of vol model is needed**

### ***Variance Is Additive with Zero Mean Assumption***

Normally, standard deviation or variance looks at the deviation from the mean. The above calculations assume a zero mean, which simplifies the calculation (typically you would expect the mean daily return to be relatively small). With a zero mean assumption, variance is additive. A mathematical proof of the formula below is given in the appendix.

Past variance + future variance = total variance

### ***Lack of Dividend Adjustment for Indices Affects Pricing***

The return calculation for a variance swap on an index does not adjust for any dividend payments that go ex. This means that the dividend modelling method can affect the pricing. Near-dated, and, hence, either known or relatively certain dividends, should be modelled discretely rather than as a flat yield. The changing exposure of the variance swap to the volatility on the ex date can be as large as 0.5 volatility points for a three year variance swap (if all other inputs are kept constant, discrete dividends lift the value of both calls and puts).

### ***Calculation Agents Might Have Discretion as to When a Market Disruption Event Occurs***

Normally the investment bank is the calculation agent for any variance swaps traded. As the calculation agent normally has some discretion over when a market disruption event occurs, this can lead to cases where one calculation agent believes a market disruption occurs and another does not. This led to a number of disputes in 2008, as it was not clear if a market or exchange disruption had occurred. Similarly if a stock is delisted, the estimate of future volatility for settlement prices is unlikely to be identical between firms, which can lead to issues if a client is long and short identical products at different investment banks. These problems are less of an issue if the counterparties are joint calculation agents.

## **HEDGING OF VARIANCE SWAPS CAN IMPACT EQUITY & VOL MARKET**

Hedging volatility, variance and gamma swaps always involve the trading of a strip of options of all strikes and delta hedging at the close. The impact the hedging of all three products has on equity and volatility markets is similar, but we shall use the term variance swaps, as it has by far the most impact of the three (the same arguments will apply for volatility swaps and gamma swaps).

### ***Short End of Volatility Surfaces Is Now Pinned to Realised***

If there is a divergence between short-dated variance swaps and realised volatility, hedge funds will put on variance swap trades to profit from this divergence. This puts pressure on the short-dated end of volatility surfaces to trade close to the current levels of realised volatility. Due to the greater risk of unexpected events, it is riskier to attempt a similar trade at the longer-dated end of volatility surfaces.

**As variance swaps are long skew, momentum of volatility trading moves skew the same way**

### ***Skew Levels Affected by Direction of Volatility Trading***

As variance swaps became a popular way to express a view of the direction of implied volatility, they impacted the levels of skew. This occurred as variance swaps are long skew (explained below) and, if volatility is being sold through variance swaps, then this weighs on skew. This occurred between 2003 and 2005, which pushed skew to a multiple year low. As volatility bottomed, the pressure from variance swap selling abated and skew recovered.

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**Direction of hedging variance swap flow, is in the direction which ensures less money is made**

### ***Delta Hedge Can Suppress or Exaggerate Market Moves***

As the payout of variance swaps is based on the close to close return, they all have an intraday delta (which is equal to zero if spot is equal to the previous day's close). As this intraday delta resets to zero at the end of the day, the hedging of these products requires a delta hedge at the cash close. A rule of thumb is that the direction of hedging flow is in the direction which makes the trade the least profit (ensuring that if a trade is crowded, it makes less money). This flow can be hundreds of millions of USD or EUR per day, especially when structured products based on selling short-dated variance are popular (as they were in 2006 and 2007, less so since the high volatility of the credit crunch).

- **Variance buying suppresses equity market moves:** If clients are net buyers of variance swaps, they leave the counterparty trader short. The trader will hedge this short position by buying a portfolio of options and delta hedging them on the close. If spot has risen over the day the position (which was originally delta neutral) has a positive delta (in the same way as a delta-hedged straddle would have a positive delta if markets rise). The end of day hedge of this position requires selling the underlying (to become delta flat), which suppresses the rise of spot. Similarly, if markets fall, the delta hedge required is to buy the underlying, again suppressing the market movement.
- **Variance selling exaggerates equity market moves:** Should clients be predominantly selling variance swaps, the hedging of these products exaggerates market moves. The argument is simply the inverse of the argument above. The trader who is long a variance swap (as the client is short) has hedged by selling a portfolio of options. If markets rise, the delta of the position is negative and, as the variance swap delta is reset to zero at the end of the day, the trader has to buy equities at the same time (causing the close to be lifted for underlyings that have increased in value over the day). If markets fall, then the trader has to sell equities at the end of the day (as the delta of a short portfolio of options is positive). Movements are, therefore, exaggerated and realised volatility increases if clients have sold variance swaps.

### ***Basis Risk Between Cash and Futures Can Cause Traders Problems***

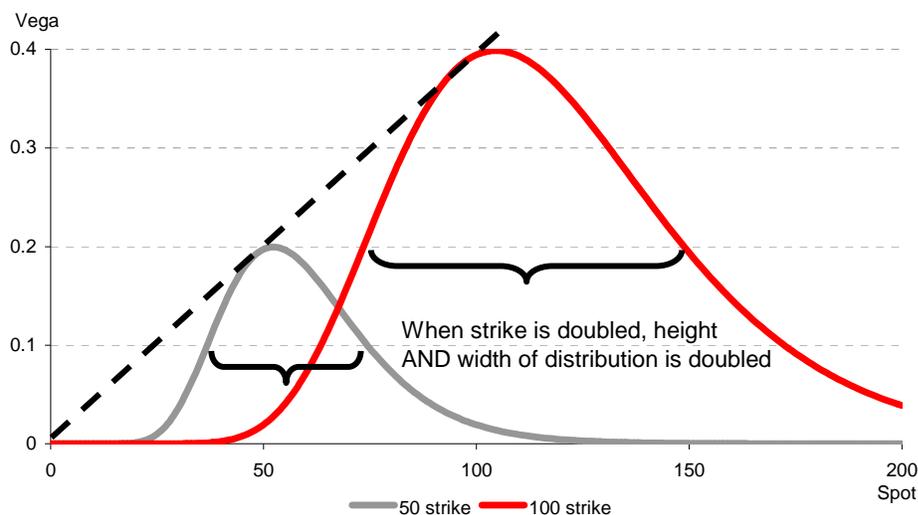
We note that the payout of variance swaps is based on the cash close, but traders normally delta hedge using futures. The difference between the cash and futures price is called the basis, and the risk due to a change in basis is called basis risk. Traders have to take this basis risk between the cash close and futures close, which can be significant as liquidity in the futures market tends to be reduced after the cash market closes.



## HEDGING VOLATILITY, VARIANCE AND GAMMA SWAPS WITH OPTIONS

As volatility, variance and gamma swaps give volatility exposure for all values of spot they need to be hedged by a portfolio of options of every strike. An equal weighted portfolio is not suitable, as the vega profile of an option increases in size and width as strike increases (ie an option of strike  $2K$  has a peak vega double the peak vega of an option of strike  $K$  and is also twice the width). This is shown below.

**Figure 1. Vega of Options of Different Strikes**



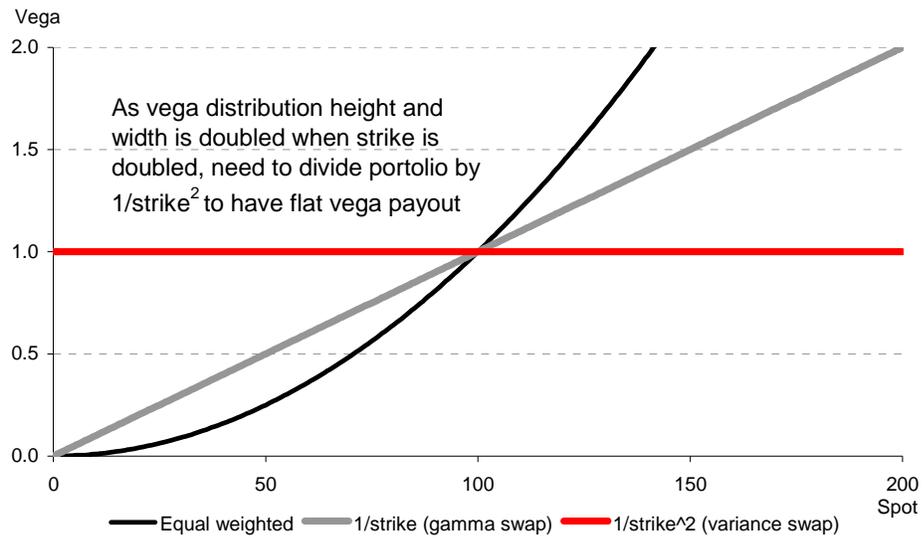
Source: Santander Investment Bolsa.

### *Variance Swaps Are Hedged with Portfolio Weighted $1/K^2$*

Because a variance swap has a flat vega profile, the correct hedge is a portfolio of options weighted  $1/K^2$  (where  $K$  is the strike of the option, ie each option is weighted by 1 divided by its own strike squared). The reason why this is the correct weighting is due to the fact the vega profile doubles in height and width if the strike is doubled. The portfolio has to be divided by strike  $K$  once, to correct for the increase in height, and again to compensate for the increase in width (for a combined weight of  $1/K^2$ ). A more mathematical proof of why the hedge for a variance swap is a portfolio of options weighted  $1/K^2$  (a so-called log contract) is given in the Appendix. As a gamma swap payout is identical to a variance swap multiplied by spot, the weighting is  $1/K$  (multiplying by spot cancels one of the  $K$ 's on the denominator). The vega profile of a portfolio weighted  $1/K$  and  $1/K^2$  is shown below, along with an equal weighted portfolio for comparison.<sup>2</sup>

**As height and width of vega profile is doubled when strike is doubled, variance swaps hedged with portfolio  $1/\text{strike}^2$**

**Figure 2. Vega of Portfolio of Options of All Strikes**



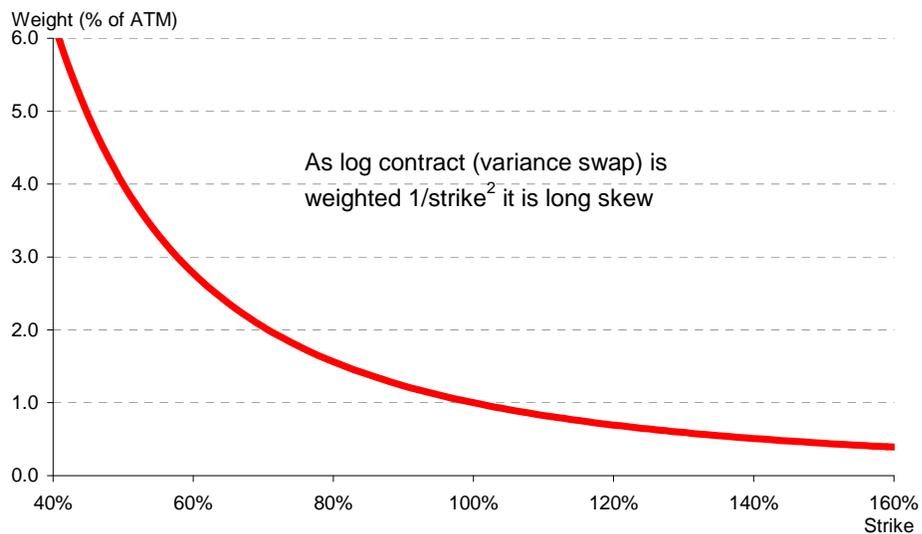
Source: Santander Investment Bolsa.

***Variance Swaps Are Long Skew and Volatility Surface Convexity***

**Variance swaps are long skew, long volatility surface convexity and vega convexity**

The  $1/K^2$  weighting means a larger amount of OTM puts are traded than OTM calls (approx 60% is made up of puts). This causes a log contract (portfolio of options weighted  $1/K^2$ ) to be long skew. The curved nature of the weighting means the wings (very out-of-the-money options) have a greater weighting than the body (near ATM options), which means a log contract is long volatility surface convexity<sup>1</sup>.

**Figure 3. Weight of Options in Log Contract (Variance Swap)**



Source: Santander Investment Bolsa.

<sup>1</sup> The inclusion of OTM (and hence convex) options mean the log contract is also long vega convexity, but they are not the same thing. Long OTM (wing) options is long vega convexity, but not volatility surface convexity (unless they are shorting the ATM or body at the same time).

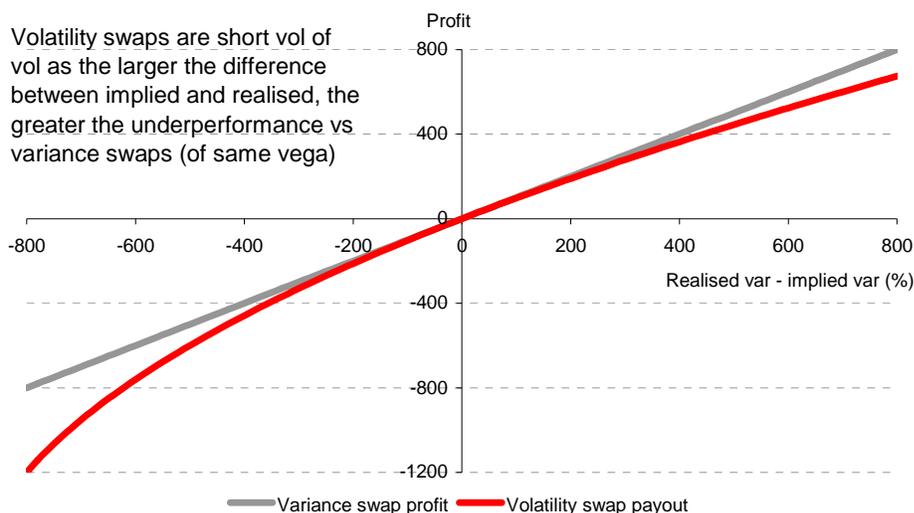


## VOLATILITY SWAPS CAN BE HEDGED WITH VARIANCE SWAPS

Unlike variance swaps (or gamma swaps) volatility swaps cannot be hedged by a static portfolio of options. Volatility swaps can be hedged with variance swaps as, for small moves, the payout can be similar (see Figure 4 below). The vega of a variance swap is equal to variance notional $\times 2\sigma$ . For example, for  $\sigma=25$  the vega is  $2\times 25 = 50$  times the size of the variance swap notional. So a volatility swap of vega “V” can be hedged with  $V/2\sigma$  variance notional of a variance swap. As a variance swap is normally quoted in vega, the vega /  $2\sigma$  formula is used to calculate the variance notional of the trade.

$$\text{Variance notional} = \text{Vega} / (2\sigma)$$

**Figure 4. Vega Profit of Variance Swap and Volatility Swap (weighted  $1/K^2$ )**



Source: Santander Investment Bolsa.

## VOLATILITY SWAPS ARE SHORT VOL OF VOL

The graph above shows the payout of a variance swap is always in excess of the payout of a volatility swap of the same vega. This is why the fair level of a variance swap is usually one or two volatility points above volatility swaps. The negative convexity of the payout (compared to a variance swap) shows that volatility swaps are short vol of vol.

A volatility swap being short vol of vol can also be shown by the fact the identical vega of a variance swap has to be weighted  $1/(2\sigma)$ . If a trader is long a volatility swap and has hedged with a short variance swap position weighted  $1/(2\sigma)$ , then as volatility decreases more variance swaps have to be sold (as  $\sigma$  decreases,  $1/(2\sigma)$  rises). Conversely, as volatility rises variance swaps have to be bought (to decrease the short). Having to sell when volatility declines and buy when it rises shows that volatility swaps are short vol of vol.

**As vega of variance swap is  $1/2\sigma$ , this shows that volatility swaps (which have constant vega) are short vol of vol**

As vol of vol and maturity are not both large at the same time, the approximation is a good one

***Difference Between Variance and Volatility Swap Prices Can Be Approximated***

Given that the difference between variance and volatility swap prices is due to the fact volatility swaps are short vol of vol, it is possible to derive the formula below, which approximates the difference between variance swap and volatility swap prices (as long as the maturity and vol of vol are not both excessive, which tends not to happen as longer maturities have less vol of vol). Using the formula, the price of a volatility swap can be approximated by the price of a variance swap less the convexity adjustment *c*. Using this formula, the difference between variance and volatility swaps is graphed in Figure 5.

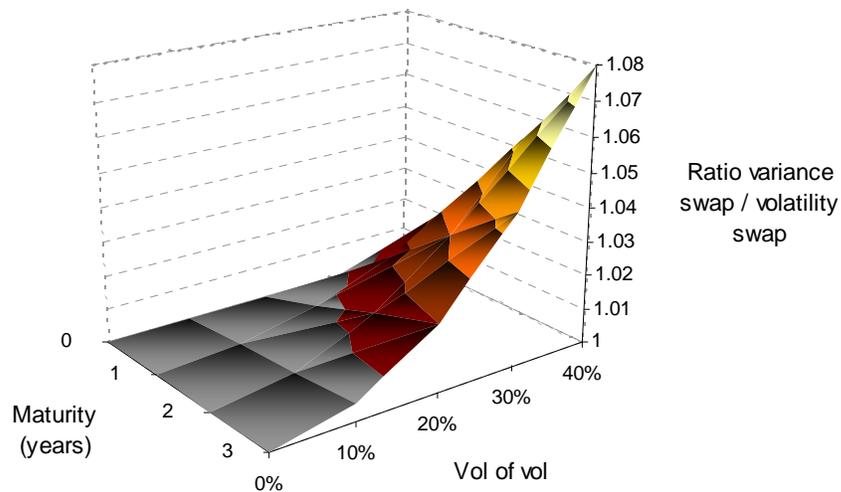
$$c \approx \frac{1}{6} \omega^2 T \sqrt{\text{variance swap price} \times e^{rT}}$$

where:

v = variance swap price

ω = volatility of volatility

**Figure 5. Difference between Variance and Volatility Swap Prices**



Source: Santander Investment Bolsa.

***Model Risk of Vol of Vol Is Small vs Tail Risk of Variance Swap***

Hedging vol of vol raises trading costs, and also introduces model risk. Since the credit crunch, however, single stock variance no longer trades and dispersion is now quoted using volatility swaps instead. Investment banks are happier taking the small model risk of vol of vol rather than being short the tail risk of a variance swap. As can be seen in the table below, variance swaps trade one or two volatility points above volatility swaps (for the most popular maturities). A simpler rule of thumb is that volatility swaps trade roughly in line with ATMf implied volatilities.



**Figure 6. Typical Values of Vol of Vol and the Effect on Variance and Volatility Swap Pricing**

Maturity	3 Month	6 Month	1 Year	2 Year
Vol of vol	85%	70%	55%	40%
Ratio var / vol	1.030	1.041	1.050	1.053
Difference var - vol (for 30% vol)	0.90	1.23	1.51	1.60

Source: Santander Investment Bolsa.

$$\text{Max Loss of Variance Swap} = \text{Swap Level} \times \text{Vega} / 2$$

The notional of a variance swap trade is  $\text{Vega} / 2\sigma_S$  ( $\sigma_S$  is traded variance swap level) and the payoff is  $(\text{realised}^2 - \sigma_S^2) \times \text{Notional}$ . The maximum loss of a variance swap is when realised variance is zero, when the loss is  $\sigma_S^2 \times \text{Notional} = \sigma_S^2 \times \text{Vega} / 2\sigma_S = \sigma_S \times \text{Vega} / 2$ .

## GREEKS OF VOLATILITY, VARIANCE AND GAMMA SWAPS

“Shadow delta”  
caused by  
correlation  
between spot and  
implied volatility

As a volatility swap needs a vol of vol model, the greeks are dependent on the model used. For variance swaps and gamma swaps, there is no debate as to the Greeks. However, practical considerations can introduce ‘shadow greeks’. In theory, a variance swap has zero delta, but in practice it has a small ‘shadow delta’ due to the correlation between spot and implied volatility (skew). Similarly, theta is not necessarily constant as it should be in theory, as movements of the volatility surface can cause it to change.

### *Variance Swap Vega Decays Linearly with Time*

As variance is additive, the vega decays linearly with time. For example, 100K vega of a one year variance swap at inception will have 75K vega after three months, 50K after six months and 25K after nine months.

### *Variance Swaps Offer Constant Cash Gamma, Gamma Swaps Have Constant Share Gamma*

Share gamma is the number of shares that need to be bought (or sold) for a given change in spot (typically 1%). It is proportional to the Black-Scholes gamma (second derivative of price with respect to spot) multiplied by spot. Cash gamma (or dollar gamma) is the cash amount that needs to be bought or sold for a given movement in spot, hence is proportional to share gamma multiplied by spot (ie proportional to Black-Scholes gamma multiplied by spot squared). Variance swaps offer a constant cash gamma (constant convexity), whereas gamma swaps offer constant share gamma (hence the name gamma swaps).

$$\gamma \times S / 100 = \text{share gamma} = \text{number of shares bought (or sold) per 1\% spot move}$$

$$\gamma \times S^2 / 100 = \text{cash (or dollar) gamma} = \text{notional cash value bought (or sold) per 1\% spot move}$$

---

## OPTIONS ON VARIANCE

As the liquidity of the variance swap market improved in the mid 2000s, market participants started to trade options on variance. As volatility is more volatile at high levels, the skew is positive (the inverse of the negative skew seen in the equity market). In addition, volatility term structure is inverted, as volatility mean reverts and does not stay elevated for long periods of time.

### OPTIONS ON VARIANCE EXPIRY = EXPIRY OF UNDERLYING VAR SWAP

An option on variance is a European option (like all exotics) on a variance swap, whose expiry is the same expiry as the option. As it is an option on variance, a volatility of volatility model is needed in order to price the option. At inception the underlying is 100% implied variance, whereas at maturity the underlying is 100% realised variance (and in between it will be a blend of the two). As the daily variance of the underlying is locked in every day, the payoff could be considered to be similar to an Asian (averaging) option.

The price of option on variance swap is quoted in volatility points (just like variance swaps)

#### *Options on Variance are Quoted in Volatility Points*

Like a variance swap the price of an option on variance is quoted in volatility points. The typical 3-month to 18-month maturity of the option is in line with the length of time it takes 3-month realised volatility to mean revert after a crisis. The poor liquidity of options on variance and the fact the underlying tends towards a cash basket over time, means a trade is usually held until expiry.

#### *Option on Variance Swap Payoff*

$$\text{Max}(\sigma_F^2 - \sigma_K^2, 0) \times \text{Variance notional}$$

where:

$\sigma_F$  = future volatility (that occurs over the life of contract)

$\sigma_K$  = strike volatility (fixed at the start of contract)

Variance notional = notional amount paid (or received) per variance point

NB: Variance notional = Vega / (2 $\sigma_S$ ) where  $\sigma_S$  = variance swap reference

### PUT CALL PARITY APPLIES TO OPTIONS ON VARIANCE

As variance swaps have a convex volatility payout, so do options on variance. As options on variance are European, put call parity applies. The fact a long call on variance and short put on variance (of the same strike) is equal to a forward on variance (or variance swap) gives the following result for options on variance whose strike is not the current level of variance swaps.

$$\text{Call Premium}_{\text{variance points}} - \text{Put Premium}_{\text{variance points}} = \text{PV}(\text{Current Variance Price}^2 - \text{Strike}^2)$$

where:

$$\text{Premium}_{\text{variance points}} = 2\sigma_S \times \text{Premium}_{\text{volatility points}} \text{ where } \sigma_S = \text{variance swap reference}$$

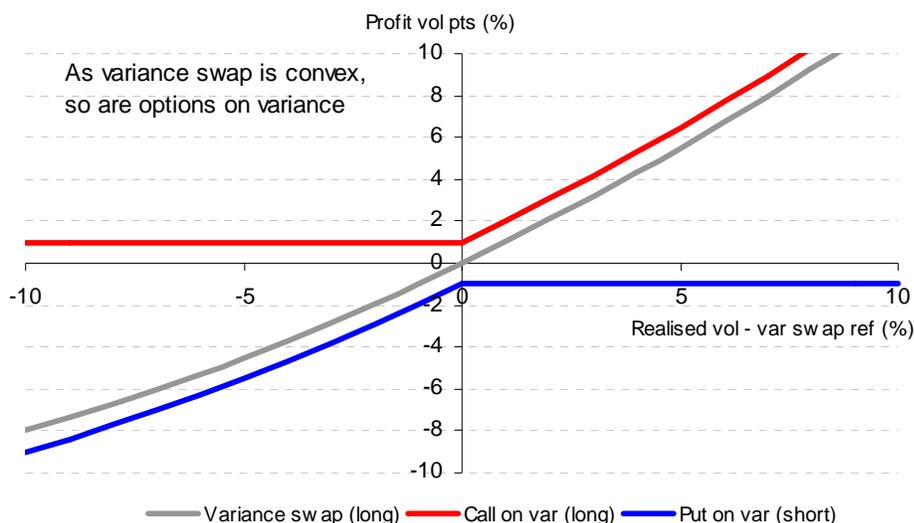


## PREMIUM PAID FOR OPTION = VEGA × PREMIUM IN VOL POINTS

The premium paid for the option can either be expressed in terms of vega, or variance notional. Both are shown below:

$$\text{Fixed leg cash flow} = \text{Variance notional} \times \text{Premium}_{\text{variance points}} = \text{Vega} \times \text{Premium}_{\text{volatility points}}$$

**Figure 7. Variance Swap, ATM Call on Variance and ATM Put on Variance**



Source: Santander Investment Bolsa.

## CONVEX PAYOUT MEANS BREAKEVENS ARE NON TRIVIAL

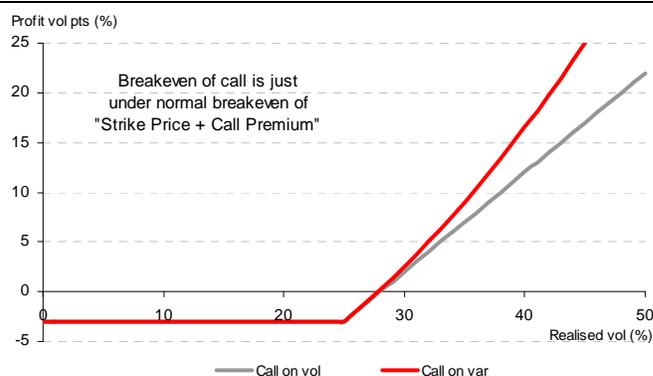
**Breakeven of options on variance is slightly below normal breakeven**

The convexity of a variance swap means that a put on a variance swap has a lower payout than a put on volatility and a call on variance swap has a higher payout than a call on volatility (see Figure 8). Similarly, it also means the maximum payout of a put on variance is significantly less than the strike. This convexity also means the breakevens for option on variance are slightly different from the breakevens for option on volatility (strike – premium for puts, strike + premium for calls).

**Figure 8. Put on Variance Swap**



**Call on Variance Swap**



Source: Santander Investment Bolsa estimates.

***Breakevens Are Similar but Not Identical to Options on Volatility***

In order to calculate the exact breakevens the premium paid (premium P in vol points × Vega) must equal the payout of the variance swap.

Premium paid = payout of variance swap

$$\text{For call option on variance: } P \times \text{Vega} = (\sigma_{\text{Call Breakeven}}^2 - \sigma_K^2) \times \frac{\text{Vega}}{2\sigma_S}$$

$$\Rightarrow \sigma_{\text{Call Breakeven}} = \sqrt{\sigma_K^2 + 2\sigma_S P} \leq \sigma_K + P = \text{Call on volatility breakeven}$$

$$\text{Similarly } \sigma_{\text{Put Breakeven}} = \sqrt{\sigma_K^2 - 2\sigma_S P} \leq \sigma_K - P = \text{Put on volatility breakeven}$$

**OPTIONS ON VARIANCE HAVE POSITIVE SKEW**

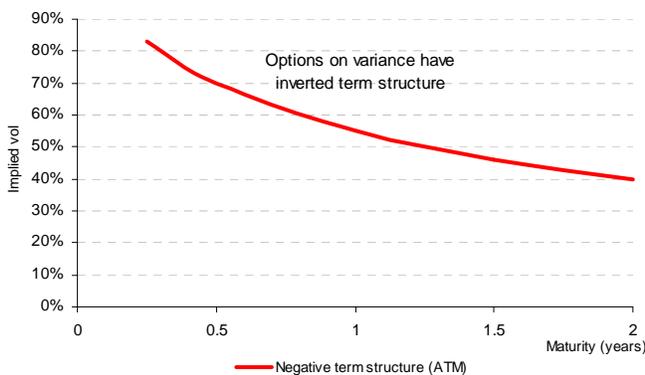
Volatility (and hence variance) is relatively stable when it is low, as calm markets tend to have low and stable volatility. Conversely volatility is more unstable when it is high (as turbulent markets could get worse with higher volatility, or recover with lower volatility). For this reason options on variance have positive skew, with high strikes having higher implied volatility than low strikes.

***Implied Variance Term Structure Is Inverted, but Not as Inverted as Realised Variance***

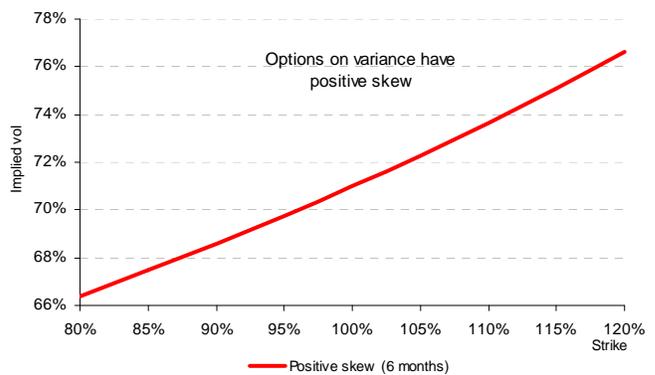
As historical volatility tends to mean revert in an 8-month time horizon (on average), the term structure of options on variance is inverted (while volatility can spike and be high for short periods of time, over the long term it trades in a far narrower range). We note that as the highest volatility occurs due to unexpected events, the peak of implied volatility (which is based on the market’s expected future volatility) is lower than the peak of realised volatility. Hence the volatility of implied variance is lower than the volatility of realised variance, especially for short maturities.

**Skew and term structure of options on variance are opposite to options on equity**

**Figure 9. Option on Variance Term Structure**



**Option on Variance Skew**



Source: Santander Investment Bolsa estimates.



## CAPPED VARIANCE SWAPS HAVE EMBEDDED OPTION ON VAR

While options on variance swaps are not particularly liquid, their pricing is key for valuing variance swaps with a cap. Capped variance swaps are standard for single stocks and emerging market indices and can be traded on regular indices as well. When the variance swap market initially become more liquid some participants did not properly model the cap, as it was seen to have little value. The advent of the credit crunch and resulting rise in volatility made the caps more valuable and now market participants fail to model them at their peril.

Variance Swap with Cap C = Variance Swap - Option on Variance with Cap C

⇒ Option on Variance with Cap C = Variance Swap - Variance Swap with Cap C

*While Value of Cap Is Small at Inception, it Can Become More Valuable as Market Moves*

A capped variance swap can be modelled as a vanilla variance swap less an option on variance, whose strike is the cap. This is true as the value of an option on variance at the cap will be equal to the difference between the capped and uncapped variance swaps. Typically the cap is at  $2.5\times$  the strike and, hence, is not particularly valuable at inception. However, as the market moves, the cap can become closer to the money and more valuable.

## OPTIONS ON VAR STRATEGIES ARE SIMILAR TO VANILLA OPTIONS

Strategies that are useful for vanilla options have a read across for options on variance. For example, a long variance position can be protected or overwritten. The increased liquidity of VIX options allows relative value trades to be put on.

Selling straddles on options on variance can also be a popular strategy as volatility can be seen to have a floor above zero. Hence, strikes can be chosen so that the lower breakeven is in line with the perceived floor to volatility.

Options on variance can also be used to hedge a volatility swap position, as an option on variance can offset the vol of vol risk embedded in a volatility swap.

Options on variance can be used as relative value trade vs VIX options or volatility swaps

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# FUTURE ON VOLATILITY INDEX

Recently futures on the VIX and vStoxx have become more liquid due to increased structured product activity. While the calculation of these volatility indices is similar to a variance swap calculation, as the payout is based on the square root of variance their payout is linear in volatility not variance. They are therefore short vol of vol, just like volatility swaps.

## PRICE IS IN BETWEEN VAR AND VOL

A future on a volatility index functions in exactly the same way as a future on an equity index. However, as the volatility index is a forward (hence linear) payout of the square root of variance, the payoff is different to a variance swap (whose payout is on variance itself). The price of a forward on a volatility index lies between the fair value of a forward volatility swap and the square root of the fair value of a forward variance swap.

$$\Rightarrow \sigma_{\text{Forward volatility swap}} \leq \text{Future on volatility index} \leq \sigma_{\text{Forward variance swap}}$$

## FUTURES ON VOLATILITY INDICES ARE SHORT VOL OF VOL

A variance swap can be hedged by delta hedging a portfolio of options (portfolio is known as a log contract, where the weight of each option is  $1/K^2$  where  $K$  is the strike). As the portfolio of options does not change, the only hedging costs are the costs associated with delta hedging. A volatility swap has to be hedged through buying and selling variance swaps (or a log contract of options); hence, it needs to have a volatility of volatility model. A variance swap is more convex than a volatility swap (as a variance swap payout is on volatility squared), a volatility swap is short convexity compared to a variance swap. A volatility swap is, therefore, short volatility of volatility (vol of vol) as a variance swap has no vol of vol risk. As the price of a future on a volatility index is linear in volatility, a future on a volatility index is short vol of vol (like volatility swaps).

### *As Vol of Vol Is Underpriced, Futures on Volatility Indices Are Overpriced*

While the price of volatility futures should be below that of forward variance swaps, retail demand and potential lack of knowledge of the client base means that they have traded at similar levels. This overpricing of volatility futures means that volatility of volatility is underpriced in these products. Being short volatility futures and long forward variance is a popular trade to arbitrage this mispricing.

## EUREX, NOT CBOE, WAS THE FIRST EXCHANGE TO LIST VOL FUTURES

While futures on the VIX (launched by the CBOE in March 2004) are the oldest currently traded, the DTB (now Eurex) was the first exchange to list volatility futures, in January 1998. These VOLAX futures were based on 3-month ATM implieds but they ceased trading in December of the same year.

Short volatility future and long forward variance is a popular trade



## STRUCTURED PRODUCTS ON VOL FUTURES IMPROVED LIQUIDITY

Initially VIX and vStoxx futures had limited liquidity. The creation of structured products has improved liquidity of these products. Similarly, the introduction of options on these futures has increased the need to delta hedge using these futures, also increasing liquidity. In the US, the size of structured products on VIX futures is so large it has moved the market.

### *Open-Ended Volatility Products on Volatility Indices Steepen Term Structure*

**Structured products often sell near-dated expiries and buy longer-dated expiries, lifting term structure**

While futures on a volatility index have the advantage of being a listed instrument, they have the disadvantage of having an expiry and, therefore, a longer-term position needs to be rolled. In response to investor demand, many investment banks sold products based on having a fixed maturity exposure on an underlying volatility index. As time passes, these banks hedge their exposure by selling a near-dated expiry and buying a far-dated expiry. The weighted average maturity is therefore kept constant but the flow puts upward pressure on the term structure. For products of sufficient size, the impact of structured products on the market ensures the market moves against them. As term structure is lifted, there is a high carry cost (as a low near-dated volatility future is sold and a high far-dated volatility future is bought).

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# APPENDIX 1. MEASURING HISTORIC VOLATILITY (AND VARIANCE)

The implied volatility (or variance) for a certain strike and expiry has a fixed value. There is, however, no single calculation for historic volatility. The number of historic days for the historic volatility calculation changes the calculation, in addition to the estimate of the drift (or average amount stocks are assumed to rise). We examine different methods of historic volatility (and variance) calculation, and demonstrate how ordinary and special dividends affect single stocks and index historic volatility.

## LOG RETURNS CAN BE APPROXIMATED BY PERCENTAGE RETURN

Volatility is defined to be the standard deviation of log returns (where return =  $P_i / P_{i-1}$ ). As returns are normally close to 1 (=100%) the log of returns is very similar to return - 1 (which is the percentage change of the price). If the return over the period is assumed to be the same for all periods, and if the mean return is assumed to be zero (it is normally very close to zero), the standard deviation of the percentage change is simply the absolute value of the percentage return. Hence, an underlying which moves 1% has a volatility of 1% for that period. As volatility is usually quoted on an annualised basis, this volatility has to be multiplied by the square root of the number of samples in a year (ie,  $\sqrt{252}$  for daily returns,  $\sqrt{52}$  for weekly returns and  $\sqrt{12}$  for monthly returns).

$$\text{Log return} = x_i = \text{Ln} \left( \frac{c_i + d_i}{c_{i-1}} \right) \text{ where } d_i = \text{ordinary (not adjusted) dividend and } c_i \text{ is close price}$$

$$\text{Volatility}^2 \text{ (not annualised)} = \sigma_x \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

where  $\bar{x}$  = drift = Average ( $x_i$ )

## BEST TO ASSUME ZERO DRIFT FOR VOLATILITY CALCULATION

The calculation for standard deviation calculates the deviation from the average log return (or drift). This average log return has to be estimated from the sample, which can cause problems if the return over the period sampled is very high or negative. As over the long term very high or negative returns are not realistic, the calculation of volatility can be corrupted by using the sample log return as the expected future return. For example, if an underlying rises 10% a day for ten days, the volatility of the stock is zero (as there is zero deviation from the 10% average return). This is why volatility calculations are normally more reliable if a zero return is assumed. In theory, the expected average value of an underlying at a future date should be the value of the forward at that date. As for all normal interest rates (and dividends, borrow cost), the forward return should be close to 100% for any reasonable sampling frequency (daily / weekly / monthly). For simplicity reasons it is easier to assume a zero log return as  $\text{Ln}(100\%) = 0$ . Assuming a zero mean return has the advantage that variance is additive, as can be seen below.

---

<sup>2</sup> We take the definition of volatility of John Hull in "Options, futures and other derivatives" in which n day volatility uses n returns and n+1 prices. We note Bloomberg uses n prices and n-1 returns.

**Volatility calculations are normally more reliable if a zero return is assumed**



## VARIANCE IS ADDITIVE IF ZERO MEAN IS ASSUMED

Frequency of returns in a year = F (e.g. 252 for daily returns)

$$\sigma_{\text{Annualised}} = \sqrt{F} \times \sigma_x = \sqrt{F} \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

As  $\bar{x} \approx 0$  if we assume zero average returns

$$\sigma_{\text{Annualised}} = \sqrt{F} \sqrt{\frac{1}{N} \sum_{i=1}^N x_i^2}$$

$$\sigma_{\text{Annualised}}^2 = \frac{F}{N} \sum_{i=1}^N x_i^2$$

Now if we assume that the total sample N can be divided up into period 1 and period 2 where period 1 is the first M returns then:

$$\sigma_{\text{Total}}^2 = \frac{F}{N_{\text{Total}}} \times \sum_{i=1}^N x_i^2$$

$$\sigma_{\text{Period 1}}^2 = \frac{F}{N_{\text{Period 1}}} \times \sum_{i=1}^M x_i^2 \text{ (where } N_{\text{Period 1}} = M\text{)}$$

$$\sigma_{\text{Period 2}}^2 = \frac{F}{N_{\text{Period 2}}} \times \sum_{i=M+1}^N x_i^2 \text{ (where } N_{\text{Period 2}} = N - M\text{)}$$

then

$$\sigma_{\text{Total}}^2 = \frac{F}{N_{\text{Total}}} \times \sum_{i=1}^N x_i^2 = \frac{F}{N_{\text{Total}}} \left( \sum_{i=1}^M x_i^2 + \sum_{i=M+1}^N x_i^2 \right)$$

$$\sigma_{\text{Total}}^2 = \frac{F}{N_{\text{Total}}} \times \sum_{i=1}^M x_i^2 + \frac{F}{N_{\text{Total}}} \times \sum_{i=M+1}^N x_i^2$$

$$\sigma_{\text{Total}}^2 = \frac{N_{\text{Period 1}}}{N_{\text{Total}}} \left( \frac{F}{N_{\text{Period 1}}} \times \sum_{i=1}^M x_i^2 \right) + \frac{N_{\text{Period 2}}}{N_{\text{Total}}} \left( \frac{F}{N_{\text{Period 2}}} \times \sum_{i=M+1}^N x_i^2 \right)$$

$$\sigma_{\text{Total}}^2 = \frac{N_{\text{Period 1}}}{N_{\text{Total}}} \sigma_{\text{Period 1}}^2 + \frac{N_{\text{Period 2}}}{N_{\text{Total}}} \sigma_{\text{Period 2}}^2$$

Hence variance is additive (when weighted by the time in each period / total time)

## APPENDIX 2. PROOF VARIANCE SWAPS CAN BE HEDGED BY LOG CONTRACT ( $1/K^2$ )

A log contract is a portfolio of options of all strikes ( $K$ ) weighted by  $1/K^2$ . When this portfolio of options is delta hedged on the close, the payoff is identical to the payoff of a variance swap. We prove this relationship and hence show that the volatility of a variance swap can be hedged with a static position in a log contract.

### PORTFOLIO OF OPTIONS WITH CONSTANT VEGA WEIGHTED $1/K^2$

In order to prove that a portfolio of options with flat vega has to be weighted  $1/K^2$ , we will define the variable  $x$  to be  $K/S$  (strike  $K$  divided by spot  $S$ ). With this definition and assuming zero interest rates, the standard Black-Scholes formula for vega of an option simplifies to:

$$\text{Vega of option} = \tau \times S \times f(x, v)$$

where

$$x = K / S \text{ (strike a ratio of spot)}$$

$$\tau = \text{time to maturity}$$

$$v = \sigma^2 \tau \text{ (total variance)}$$

$$f(x, v) = \frac{1}{\sqrt{2\pi}} \times e^{-\frac{d_1^2}{2}}$$

$$d_1 = \frac{\text{Ln}\left(\frac{1}{x}\right) + \frac{v}{2}}{\sqrt{v}}$$

If we have a portfolio of options where the weight of each option is  $w(K)$ , then the vega of the portfolio of options  $V(S)$  is:

$$V(S) = \tau \int_{K=0}^{\infty} w(K) \times S \times f(x, v) dK$$

As  $K = xS$  this means  $dK / dx = S$ , hence  $dK = S \times dx$  and we can change variable  $K$  for  $x$ .

$$V(S) = \tau \int_{x=0}^{\infty} w(xS) \times S^2 \times f(x, v) dx$$

In order for the portfolio of options to have a constant vega – no matter what the level of spot –  $dV(S)/dS$  has to be equal to zero.

$$\frac{dV}{dS} = \tau \int_{x=0}^{\infty} \frac{d}{dS} [S^2 w(xS)] \times f(x, v) dx = 0$$



And by the chain rule:

$$\Rightarrow \tau \int_{x=0}^{\infty} \left[ 2Sw(xS) + S^2 \frac{d}{dS} w(xS) \right] \times f(x, v) dx = 0$$

$$\Rightarrow \tau \int_{x=0}^{\infty} S \left[ 2w(xS) + S \frac{d}{dS} w(xS) \right] \times f(x, v) dx = 0$$

As  $d/dS = (d/dK) \times (dK/dS)$ , and  $dK/dS = x$

$$\Rightarrow \tau \int_{x=0}^{\infty} S \left[ 2w(xS) + xS \frac{d}{dK} w(xS) \right] \times f(x, v) dx = 0$$

As  $K = xS$

$$\Rightarrow \tau \int_{x=0}^{\infty} S \left[ 2w(xS) + K \frac{d}{dK} w(K) \right] \times f(x, v) dx = 0$$

$$\Rightarrow 2w + K \frac{d}{dK} w(K) = 0 \text{ for all values of } S$$

$$\Rightarrow w(K) = \frac{\text{constant}}{K^2}$$

---

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Rating	Definition	% of Companies	
		Covered with This Rating	Provided with Investment Banking Services in Past 12M
<b>Buy</b>	Upside of more than 15%.	45.10	26.09
<b>Hold</b>	Upside of 10%-15%.	32.68	39.13
<b>Underweight</b>	Upside of less than 10%.	13.07	34.78
<b>Under Review</b>		0.00	0.00

NOTE: Given the recent volatility seen in the financial markets, the recommendation definitions are only indicative until further notice.

(\*) Target prices set from January to June are for December 31 of the current year. Target prices set from July to December are for December 31 of the following year.

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 Fax: 571-592-0638

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Tel: 54114-341-1052  
 Fax: 54114-341-1226

#### Caracas

Tel: 582-401-4306  
 Fax: 582-401-4219

#### Lima

Tel: 511-215-8100  
 Fax: 511-215-8185

#### Mexico City

Tel: 5255-5629-5040  
 Fax: 5255-5629-5846

#### Santiago

Tel: 562-336-3300  
 Fax: 562-697-3869

#### São Paulo

Tel: 5511-5538-8226  
 Fax: 5511-5538-8407

#### Tokyo

Tel: 813-3211-0356  
 Fax: 813-3211-0362